LETTERS

Feynman's Dining Hall Dynamics

Richard Feynman's adventurous autobiography *"Surely You're Joking, Mr. Feynman!"* (Norton, New York, 1985) is one of the books I enjoyed most in recent years. In it is this story: "I was in the [Cornell] cafeteria and some guy, fooling around, throws a plate in the air. As the plate went up in the air I saw it wobble, and I noticed the red medallion of Cornell on the plate going around. It was pretty obvious to me that the medallion went around faster than the wobbling.... I started to figure out the motion of the rotating plate. I discovered that when the angle is very slight, the medallion rotates twice as fast as the wobble rate-two to one. It came out of a complicated equation!" He then tried to look at this in a more fundamental way. "I don't remember how I did it, but I ultimately worked out what the motion of the mass particles is, and how all the accelerations balance to make it come out two to one." He showed this to Hans Bethe, who remained. unimpressed. But this eventually rekindled his love for "playing" with physics, and "the diagrams and the whole business that I got the Nobel Prize for came from that piddling around with the wobbling plate."

Great story, except for one little twist: A torque-free plate *wobbles* twice as fast as it *spins* when the wobble angle is slight. The ratio of spin to wobble rates is 1:2, not 2:1!

Being less adventurous, I can only present the reasoning in a conventional manner through the princi ple of angular momentum conservation. There are a number of ways to do this. The simplest is perhaps to solve Euler's equation for an axial- symmetric rigid body (a "top") in the body reference frame. Let the top's three principal moments of inertia be A, A and C, where $C \neq A$ so that rotation about the figure axis is stable and a wobble is possible. For small wobble angles, the solution to Euler's equation gives the wobble rate $\omega = f\Omega$, where $f \equiv (C - A)/A$ and Ω is the spin rate. The value of f is always between -1 and 1, approaching -1 for a slender cylinder (a "rod," for which $C = 0$) and 1 for at thin circular disk (a "plate," for which $C = 2A$). For a slightly oblate spheroid (such as the Earth or a neutron star), f is slightly larger than 0 and the wobble is very slow compared with the spin. In any case, we have $-\Omega < \omega < \Omega$.

But this is as seen from the body frame, which is rotating. In the inertial frame (where Feynman looks on as the Cornell plate goes up in the air) the wobble rate is $\omega_0 = \omega + \Omega$, so that $0 < \omega_0 < 2\Omega$. For a rod, $\omega_0 = 0$ (it does not wobble); for the Earth, w_0 is slightly faster than one cycle every 24 hours. And for our plate, $w_0 = 2\Omega$: The spin-to-wob- ble ratio is 1:2.

"Surely You're Joking, Mr. Feyn- man!" has little to do with physics *per se*, and the above story is one of the few mentions in the book about specif- ic physics. Whether the error is a mere slip in memory, or, in keeping with the spirit of the author and the book, another practical joke meant for those who do physics without experimenting, we do not know and perhaps never will. One thing is certain, though: This story appears on page 157 of the book and the text is 314 pages long; and we all know that the ratio of 157 to 314 is 1:2!

Reference

L.D. Landau, E.M. Lifshitz, Mechanics, 3rd ed.. Pergamon, Oxford (1976).

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Feynman: Wobbles, Bottles and Ripples

Being more adventurous but less careful than B. Fong Chao (February 1989, page 15), I have tried to reconstruct Richard Feynman's explanation for the motion of a wobbling spinning plate. Why, in "simple" terms, does a wobbling plate wobble twice as fast as it spins? One seeks an explanation like Feynman's textbook explanation for the torque on a forced-precession gyroscope in terms of the Coriolis acceleration of its particle masses. The wobbling plate is in free precession and, it turns out, is in some sense easier to understand. So here, for general entertainment, is an explanation with some equations to help with visualization.

Consider a particle in a circular orbit about the origin that is slightly tilted off a reference plane. Consider another particle of equal mass also in circular orbit about the origin, but on a plane tilted just slightly the other way. Looking down at the reference plane, let the particles be one-quarter of a revolution apart. Specifically, say $\boldsymbol{r}_1 \approx (\cos t, \sin t, \epsilon \cos t)$ and $\boldsymbol{r}_2 \approx (\sin t, -\cos t, -\epsilon \sin t)$. These two particles, with the origin, define a plane. One can see that this plane wobbles around twice as fast as either of the particles by tracing the particles' motion with two fingers (a quarter of an orbit should give the idea). That is, the x and y components of its downward normal are $\epsilon \cos 2t$ and $\epsilon \sin 2t$. Each of the planar circular orbits could be caused, say, by tying each of the particles to the origin with a massless rod. But since the angle between these two particles is always $\pi/2$ (to first order), the two rods might as well be welded to each. other, though still hinged at the origin. The two connected particles. now constitute a rigid body with an inertia tensor about the origin proportional to that of any planar axisymmetric body about its center of mass– a plate, for example. The particle pair's equations of rotational motion and its actual rotational motion are thus the same as those of a freely moving plate.

So it turns out that the slight. wobbling of a free-flying plate is in fact a very simple motion kinemaically. All of the particles are traveling in circles (almost) around the center. All particles on a given radial line share an orbital plane, tilted slightly from the planes of other radial lines. This kinematics has other consequences as well. There exists the possibility that a planetary ring of particles in independent circular orbits could appear as a rigid wobbling "hula hoop." Also, a loop of chain floating in space could move in this rigid mode even though the chain has no bending stiffness.

There are other problems for which it is useful to realize that the rotational motion of any three-dimensional rigid body is totally equivalent to that of three particles attached to three rigid massless rods that are welded orthogonally to one another and pivoted at the origin (just two particles for flat objects). Or, if one does not like tying things to a fixed origin, one can weld three dumbbells together to construct an object that looks like a child's jack (six masses).

Rigid-body dynamics is hard in general because it is hard to figure the interaction forces and moments that might maintain the rigid-body constraint, even with the few-particle descriptions of a rigid body described above. But Feynman's wobbling plate problem just happens to be simple in this regard.

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